GVP COLLEGE OF ENGINEERING FOR WOMEN (A)

II BTech - I sem - Reg - Nov - 2025

DISCRETE MATHEMATICAL STRUCTURES

Sub Code: 24BM11RC05

Scheme of Evaluation

1a) Reflexive : $x-x=0$ is an integer — 2M

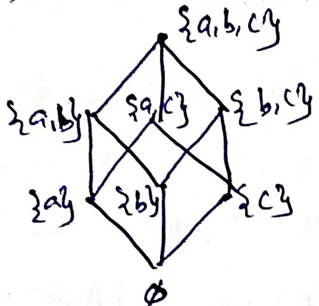
Symmetry : $x-y$ & $y-x$ integers — 2M

Transitivity : $x-y$ & $y-z$ are integers — 2M

$\Rightarrow x-z=(x-y)-(y-z)$ is also integer

$\therefore$ It is equivalence by above three — 1M

1b) $P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ — 2M



— 5M

2a) i) Def : $R^{+} = R \cup R^2 \cup R^3 \cup \cdots R^n$ — 2M

Example : Any example — 1M

2a) ii) one - one $x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$ — 1M

onto $f: X \to Y \Rightarrow$ Range of $f = Y$ — 1M

Bijection one - one & onto — 1M

Identity : $f(x) = x$ — 1M

2b) $P(2) = \lfloor \frac{100}{2} \rfloor = 50$, $P(3) = \lfloor \frac{100}{3} \rfloor = 33$, $P(5) = \lfloor \frac{100}{5} \rfloor = 20$

$P(2 \& 3) = \lfloor \frac{100}{6} \rfloor = 16$, $P(2 \& 5) = \lfloor \frac{100}{10} \rfloor = 10$, $P(3 \& 5) = \lfloor \frac{100}{15} \rfloor = 6$ } — 3M

$P(2 \& 3 \& 5) = \lfloor \frac{100}{30} \rfloor = 3$

$\therefore P(2 \cup 3 \cup 5) = (50+33+20) - (16+10+6) + 3 = 74$ — 2M

$P(\overline{2 \cup 3 \cup 5}) = 100 - 74 = 26$ — 2M

3a) i)

$P \to (Q \to R) \Leftrightarrow P \to (\neg Q \vee R)$ — $\frac{1}{2}$M

$\Leftrightarrow \neg P \vee (\neg Q \vee R) \Leftrightarrow (\neg P \vee \neg Q) \vee R$ — 1M

$\Leftrightarrow \neg (P \wedge Q) \vee R$ — $\frac{1}{2}$M

$\Leftrightarrow (P \wedge Q) \to R$ — 1M

3a ii)

$[P \wedge Q \wedge (R \vee \neg R)] \vee [\neg P \wedge R \wedge (Q \vee \neg Q)] \vee [Q \wedge R \wedge (P \vee \neg P)]$ — 2M

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q) \vee (\neg P \wedge R \wedge \neg Q) \vee (Q \wedge R \wedge P) \vee (Q \wedge R \wedge \neg P)$ — 1M

3b) E: Jack misses many classes through illness ; A: Jack reads lot of books ;
S: Jack fails high school ; H: Jack is uneducated — 2M

Given $E \to S, S \to H, A \to \neg H, E \wedge A$ — 1M

1	$E \to S$	Rule P	
2	$S \to H$	Rule P	
3	$E \to H$	Rule T, 1 & 2 : $P \to Q, Q \to R \Rightarrow P \to R$	
4	$A \to \neg H$	Rule P	
5	$H \to \neg A$	Rule T, 4 ; $P \to Q \Leftrightarrow \neg Q \to \neg P$	
6	$E \to \neg A$	Rule T, 3 & 5, $P \to Q, Q \to R \Rightarrow P \to R$	— 3M
7	$\neg E \vee \neg A$	Rule T, 6, $P \to Q \Leftrightarrow \neg P \vee Q$	
8	$\neg (E \wedge A)$	Rule T, 7, $\neg (P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$	
9	$E \wedge A$	Rule P	
10	F	Rule T ; 8, 9 ; $\neg P \wedge P \Leftrightarrow F$	

Thus, the given set of premises leads to a contradiction and hence it is inconsistent — 1M

4a)

1.	$\neg R \vee P$	Rule P	— 2M
2.	R	Rule P (additional premise)	
3.	P	Rule T 1 & 2 ; $P, \neg P \vee Q \Rightarrow Q$	
4.	$P \to (Q \to S)$	Rule P	
5.	$Q \to S$	Rule T, 3 & 4, $P, P \to Q \Rightarrow Q$	— 5M
6.	Q	Rule P	
7.	S	Rule T, 5 & 6, $P \to Q, P \Rightarrow Q$	
8.	$R \to S$	Rule CP ; 2 & 7	

4b) Let $L(x)$: x is a living thing ; $P(x)$: x is a plant ; $A(x)$: x is an animal ; $H(x)$: x have heart ; let y represents David's dog — (2M)

$\forall x(L(x)\to(P(x)\vee A(x)))$, $L(y)\wedge\sim P(y)$, $\forall x(A(x)\to H(x)) \Rightarrow H(y)$ — (1M)

1.	$\forall x(L(x)\to(P(x)\vee A(x)))$	Rule P
2.	$L(y)\wedge\neg P(y)$	Rule P
3.	$\forall x(A(x)\to H(x))$	Rule P
4.	$L(y)\to(P(y)\vee A(y))$	Rule T, 1, US
5	$A(y)\to H(y)$	Rule T, 3, US
6.	$L(y)$	Rule T, 2, $P\wedge Q\Rightarrow P$
7	$\sim P(y)$	Rule T, 2, $P\wedge Q\Rightarrow Q$
8.	$P(y)\vee A(y)$	Rule T, 4, 6 ; $P\to Q, P\Rightarrow Q$
9.	$A(y)$	Rule T, 7, 8 ; $\neg P, P\vee Q\Rightarrow Q$
10	$H(y)$	Rule T, 5, 9 : $P\to Q, P\Rightarrow Q$

(4M)

5a) $P(1) \Rightarrow 1=1^2$ — (1M)

Assume for $P(K)$: $1+3+5+\cdots+(2K-1)=K^2$ — (2M)

$P(K+1) = 1+3+5+\cdots+(2K-1)+(2K+1)$ — (2M)

$= K^2+2K+1$ — (1M)

$=(K+1)^2$ — (1M)

5b) $662 = 414\times1+248$ — (1M)

$414 = 248\times1+166$ — (1M)

$248 = 166\times1+82$ — (1M)

$166 = 82\times2+0$ — (1M)

$GCD = 2$ — (1M)

Euclidean algorithm — (2M)

$GCD(a,b)=GCD(a,r_1)=\cdots$

$=GCD(r_n,0)=r_n$

6a) $m_1=3, m_2=4, m_3=5 \Rightarrow GCD(3,4)=GCD(3,5)=GCD(4,5)=1$ — (1M).

$a_1=2, a_2=3, a_3=2$

$M=m_1\times m_2\times m_3=60.$

$M_1=\frac{M}{m_1}=20, \quad M_2=\frac{M}{m_2}=15, \quad M_3=\frac{M}{m_3}=12$ } — (2M)

$M_iy_i\equiv1 \pmod{m_i} \Rightarrow y_1=2, y_2=3, y_3=3$ — (2M)

$x=\sum a_iM_iy_i = 287$ — (1M)

$x=287 \pmod{60} \Rightarrow x\equiv47 \pmod{60}$ — (1M)

6b) Statement $a^{p-1} \equiv 1 \pmod{p}$ where p is prime — (2M)

$7^{12} \equiv 1 \pmod{13}$ — (1M)

$(7^{12})^{18} \equiv 1^{18} \pmod{13}$ — (1M)

$7^{216} \equiv 1 \pmod{13}$ — (1M)

$7^{222} \equiv 7^{6} \pmod{13}$ — (1M)

$7^{2} \equiv 49 \equiv 10 \pmod{13}$

$7^{6} \equiv 12 \pmod{13}$ — (1M)

Ans 12

7a) $r^2 - 3r - 2 = 0$ (1M) $\Rightarrow r_1 = \frac{3+\sqrt{17}}{2}$, $r_2 = \frac{3-\sqrt{17}}{2}$ — (2M)

$a_n = c_1 r_1^n + c_2 r_2^n$ — (1M)

For solving $c_1 = \frac{\sqrt{17}+17}{34}$ — (1M)

& $c_2 = \frac{17-\sqrt{17}}{34}$ — (1M)

$a_n = c_1 r_1^n + c_2 r_2^n$ (Substituting) — (1M)

7b) $a_1 = a_0 + 1$ — (1M)

$a_2 = a_1 + 2 = a_0 + 1 + 2$ — (1M)

$a_3 = a_2 + 3 = a_0 + 1 + 2 + 3$ — (1M)

$\vdots$

$a_n = a_0 + (1 + 2 + \cdots n)$ — (2M)

$a_n = 2 + n(n+1)/2$ — (2M)

8a) $\sum_{n=2}^{\infty} a_n x^n - 7 \sum_{n=2}^{\infty} a_{n-1} x^n + 10 \sum_{n=2}^{\infty} a_{n-2} x^n = 0$ — (2M)

$\therefore A(x) - a_0 - a_1 x - 7\left[x A(x) - a_0 x\right] + 10x^2 A(x) = 0$ — (2M)

$A(x) = \frac{a_0 + (a_1 - 7a_0)x}{1 - 7x + 10x^2}$ — (2M)

$A(x) = \frac{C}{1-5x} + \frac{D}{1-2x}$ — (2M)

$a_n = C \cdot 5^n + D \cdot 2^n$ — (2M)

8b) Each coin toss has 2 possible outcomes ⇒ H & T — (2M)

For this 2 outcomes.

Like this for 10 coins, $2 \times 2 \times \cdots \times 2 = 2^{10}$ — (2M)

(10 times)

9a) Two graphs G & G' are isomorphic if there is a function $f: V(G) \to V(G')$ such that

(i) f is one-one

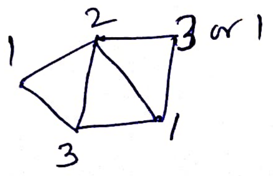
(ii) f is onto and

(iii) For each pair of vertices u and v of G, $\{u, v\} \in E(G)$ iff $\{f(u), f(v)\} \in E(G')$ i.e., f preserves adjacency

— (3M)

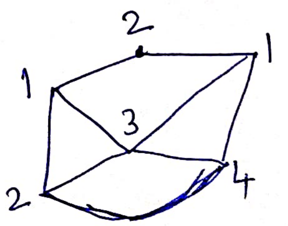
Eg: for any example and explanation — (4M)

9b)i) Chromatic number is the minimum number of colours needed to color the vertices of a graph such that no two adjacent vertices have the same color. — (1M)



Ans: 3 colors

(1M)



Ans: 4 colors

(1M)

(ii)

Euler graph	Hamiltonian graph	
Traversing every edge exactly once	visiting every vertex exactly once.	— (2M)
All vertices must have an even degree for a circuit	No simple condition exists	— (2M)

10)a) BFS : This graph explores level by level. It starts at a source and then visits all its immediate neighbors. — (2M)

DFS : explores as deeply as possible along each branch before backtracking. It starts at a source node, then explores one of its unvisited neighbors, then one of that node's unvisited neighbors, and so on, until it reaches a node with no unvisited neighbors. — (2M)

BFS — Example $(1\frac{1}{2}M)$

DFS — Example $(1\frac{1}{2}M)$

10b) Starts at v_1.

Add $v_1 \xrightarrow{6} v_2$

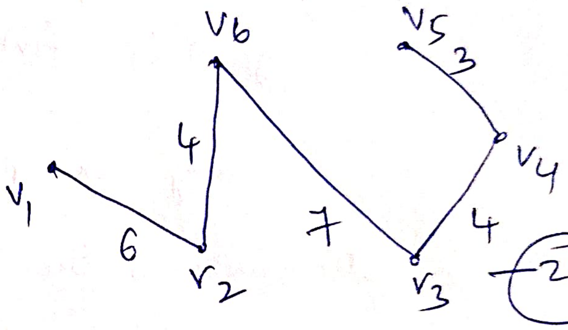
Add $v_2 \xrightarrow{4} v_6$

Add $v_6 \xrightarrow{7} v_3$

Add $v_3 \xrightarrow{4} v_4$

Add $v_4 \xrightarrow{3} v_5$

— (4M)



— (2M)

Weight of the graph $= 6+4+7+4+3$

$= 24$ — (1M)

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